

CoE 163

Computing Architectures and Algorithms

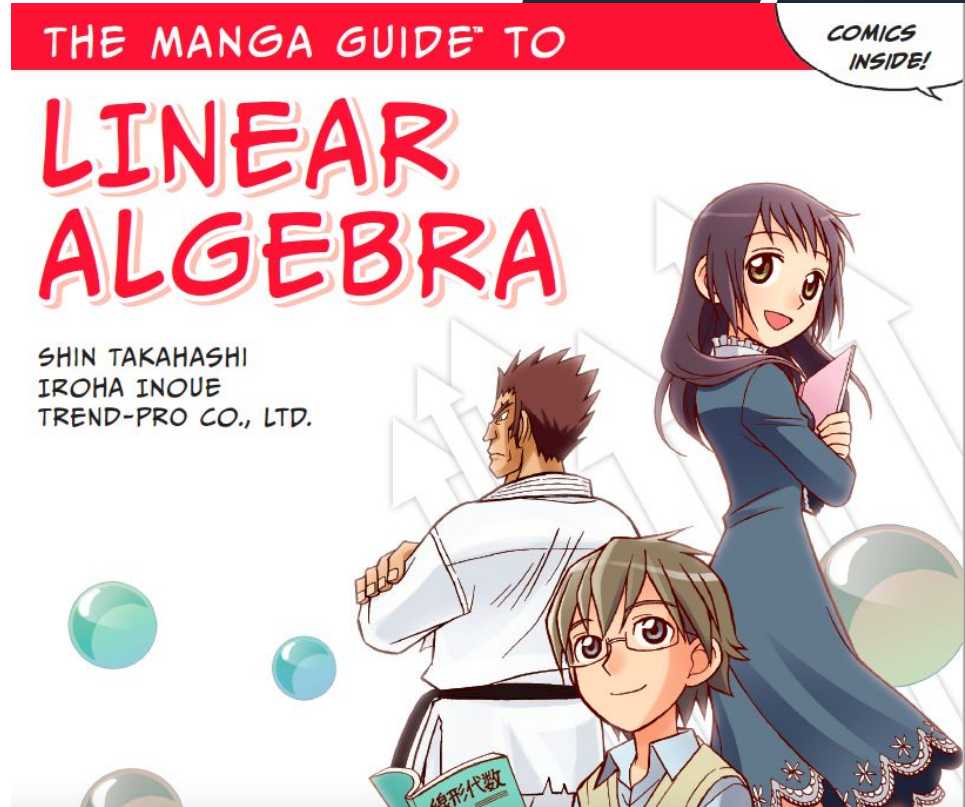
Review of Linear Algebra Operations

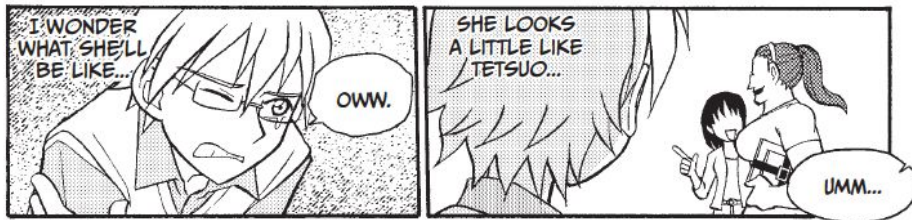
What is linear algebra?



Would like to acknowledge and thank
this book:

Takahashi S, Inoue I, Lindh
F, Co T. 2012. The Manga
Guide to Linear Algebra. No
Starch Press (Manga guide
series).





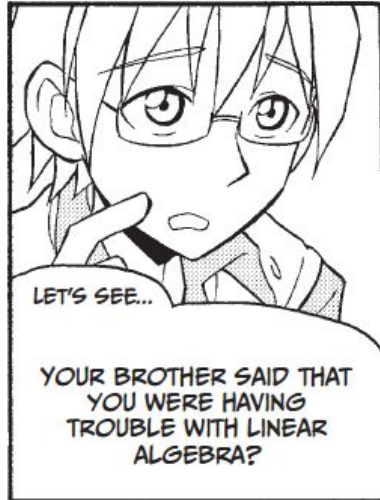
Reiji Yurino was asked by his karate sensei to tutor the sensei's sister, Misa Ichinose

Taken from: Takahashi S, Inoue I, Lindh F, Co T. 2012. The Manga Guide to Linear Algebra. No Starch Press (Manga guide series).

AN OVERVIEW OF
LINEAR ALGEBRA

WELL THEN, WHEN
WOULD YOU LIKE
TO START?

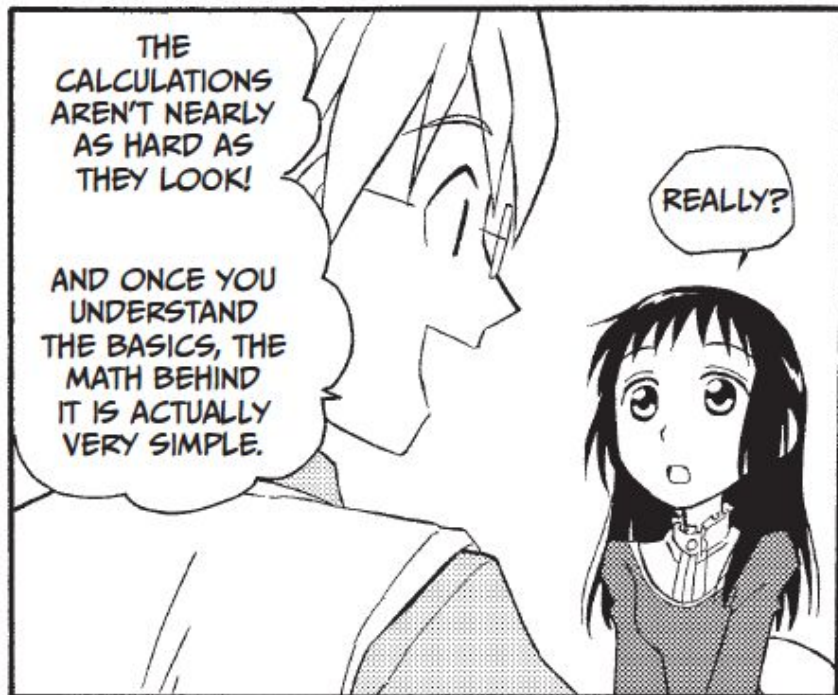
HOW ABOUT
RIGHT NOW?



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2012. The Manga Guide to Linear Algebra. No
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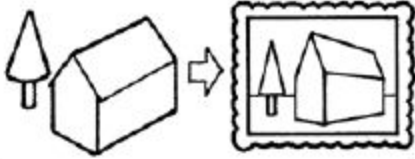


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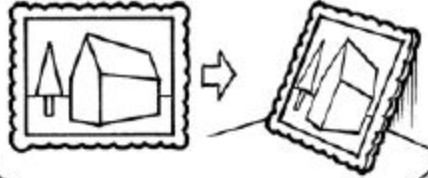


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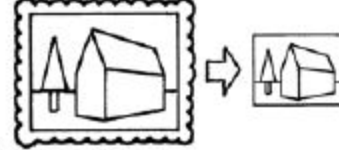
FROM THREE TO TWO DIMENSIONS



FROM TWO TO THREE DIMENSIONS



FROM TWO TO THE SAME TWO DIMENSIONS



BROADLY SPEAKING, LINEAR ALGEBRA IS ABOUT TRANSLATING SOMETHING RESIDING IN AN m -DIMENSIONAL SPACE INTO A CORRESPONDING SHAPE IN AN n -DIMENSIONAL SPACE.

OH!



Taken from: Takahashi S, Inoue I, Lindh F, Co T. 2012. The Manga Guide to Linear Algebra. No Starch Press (Manga guide series).

Linear Algebra

- Branch of mathematics that deals with linear equations
 - Representation in vector spaces
 - Representation in matrices



Typical application: solving linear systems of equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1N}x_N = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2N}x_N = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3N}x_N = b_3$$

...

$$a_{M1}x_1 + a_{M2}x_2 + a_{M3}x_3 + \cdots + a_{MN}x_N = b_M$$

- N unknowns: $x_j, j = 1, 2, \dots, N$
- M equations
- Known numbers:
 - Coefficients $a_{ij}, i = 1, 2, \dots, M$ and $j = 1, 2, \dots, N$
 - $b_i, i = 1, 2, \dots, M$



Typical application: solving linear systems of equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1N}x_N = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2N}x_N = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3N}x_N = b_3$$

...

$$a_{M1}x_1 + a_{M2}x_2 + a_{M3}x_3 + \cdots + a_{MN}x_N = b_M$$

- Can be written in matrix form

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$$



Matrix representation of linear system of equations

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$$

Matrix of coefficients, \mathbf{A}

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M1} & a_{M2} & \cdots & a_{MN} \end{bmatrix}$$

Unknowns, \mathbf{x}
column vector with
 N entries

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_N \end{bmatrix}$$

Right-hand side, \mathbf{b}
column vector with
 M entries

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_M \end{bmatrix}$$

Operations that we will focus on in this course

- Matrix-matrix multiplication
- Gaussian elimination
- Matrix inversion
- Matrix decomposition



Matrix-Matrix Multiplication



$$\text{THE PRODUCT } \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} = \begin{pmatrix} 1x_1 + 2x_2 & 1y_1 + 2y_2 \\ 3x_1 + 4x_2 & 3y_1 + 4y_2 \\ 5x_1 + 6x_2 & 5y_1 + 6y_2 \end{pmatrix}$$

CAN BE DERIVED BY TEMPORARILY SEPARATING THE
TWO COLUMNS $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ AND $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, FORMING THE TWO PRODUCTS

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1x_1 + 2x_2 \\ 3x_1 + 4x_2 \\ 5x_1 + 6x_2 \end{pmatrix} \quad \text{AND} \quad \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1y_1 + 2y_2 \\ 3y_1 + 4y_2 \\ 5y_1 + 6y_2 \end{pmatrix}$$

AND THEN REJOINING THE RESULTING COLUMNS:

$$\begin{pmatrix} 1x_1 + 2x_2 & 1y_1 + 2y_2 \\ 3x_1 + 4x_2 & 3y_1 + 4y_2 \\ 5x_1 + 6x_2 & 5y_1 + 6y_2 \end{pmatrix}$$

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AS YOU CAN SEE FROM THE EXAMPLE BELOW,
CHANGING THE ORDER OF FACTORS USUALLY
RESULTS IN A COMPLETELY DIFFERENT PRODUCT.



$$\cdot \begin{pmatrix} 8 & -3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 8 \cdot 3 + (-3) \cdot 1 & 8 \cdot 1 + (-3) \cdot 2 \\ 2 \cdot 3 + 1 \cdot 1 & 2 \cdot 1 + 1 \cdot 2 \end{pmatrix} = \begin{pmatrix} 24 - 3 & 8 - 6 \\ 6 + 1 & 2 + 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 7 & 4 \end{pmatrix}$$

$$\cdot \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 8 & -3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 \cdot 8 + 1 \cdot 2 & 3 \cdot (-3) + 1 \cdot 1 \\ 1 \cdot 8 + 2 \cdot 2 & 1 \cdot (-3) + 2 \cdot 1 \end{pmatrix} = \begin{pmatrix} 24 + 2 & -9 + 1 \\ 8 + 4 & -3 + 2 \end{pmatrix} = \begin{pmatrix} 26 & -8 \\ 12 & -1 \end{pmatrix}$$

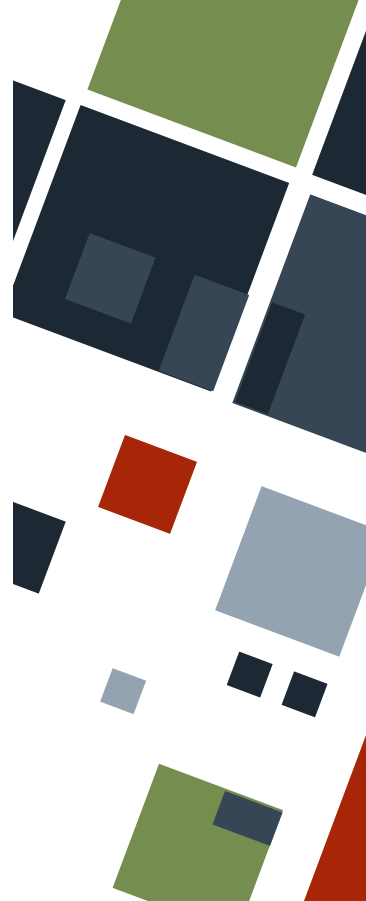
AND YOU HAVE TO WATCH OUT.



$$\begin{matrix} & 1 & 2 & \cdots & n \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} & \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix} & \begin{matrix} 1 \\ 2 \\ \vdots \\ n \end{matrix} \end{matrix}$$

AN $m \times n$ MATRIX TIMES AN $n \times p$ MATRIX YIELDS AN $m \times p$ MATRIX.

MATRICES CAN BE MULTIPLIED ONLY IF THE NUMBER OF COLUMNS IN THE LEFT FACTOR MATCHES THE NUMBER OF ROWS IN THE RIGHT FACTOR.



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Gaussian Elimination

CALCULATING INVERSE MATRICES

COFACTOR
METHOD

GAUSSIAN
ELIMINATION

THERE ARE TWO MAIN WAYS TO
CALCULATE AN INVERSE MATRIX:

USING COFACTORS OR USING
GAUSSIAN ELIMINATION.

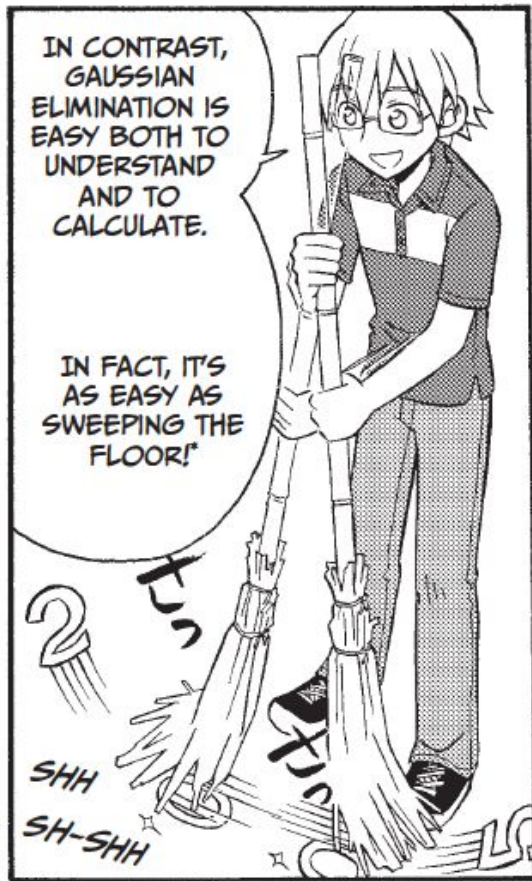
THE CALCULATIONS
INVOLVED IN THE
COFACTOR METHOD CAN
VERY EASILY BECOME
CUMBERSOME, SO...

~~COFACTOR METHOD~~

IGNORE IT AS LONG AS
YOU'RE NOT EXPECTING
IT ON A TEST.

CAN
DO.

Taken from: Takahashi S, Inoue I, Lindh F, C
2012. The Manga Guide to Linear Algebra.
Starch Press (Manga guide series).



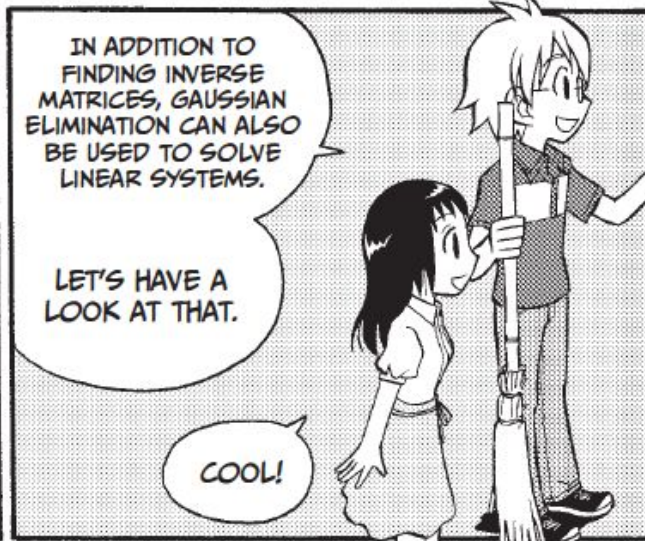
IN CONTRAST,
GAUSSIAN
ELIMINATION IS EASY BOTH TO
UNDERSTAND
AND TO
CALCULATE.

IN FACT, IT'S
AS EASY AS
SWEEPING THE
FLOOR!



ANYWAY, I WON'T TALK
ABOUT COFACTORS AT
ALL TODAY.

GOTCHA.



IN ADDITION TO
FINDING INVERSE
MATRICES, GAUSSIAN
ELIMINATION CAN ALSO
BE USED TO SOLVE
LINEAR SYSTEMS.

LET'S HAVE A
LOOK AT THAT.

COOL!

* THE JAPANESE TERM FOR GAUSSIAN ELIMINATION IS *HAKIDASHIHOU*, WHICH ROUGHLY TRANSLATES TO "THE SWEEPING OUT METHOD." KEEP THIS IN MIND AS YOU'RE READING THIS CHAPTER!

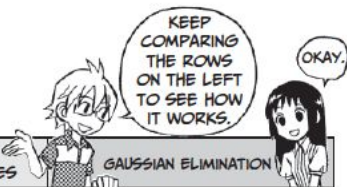
Taken from: Takahashi S, Inoue I, Lindh F, C
2012. The Manga Guide to Linear Algebra.
Starch Press (Manga guide series).

PROBLEM

Solve the following linear system:

$$\begin{cases} 3x_1 + 1x_2 = 1 \\ 1x_1 + 2x_2 = 0 \end{cases}$$

SOLUTION



THE COMMON METHOD	THE COMMON METHOD EXPRESSED WITH MATRICES	GAUSSIAN ELIMINATION
$\begin{cases} 3x_1 + 1x_2 = 1 \\ 1x_1 + 2x_2 = 0 \end{cases}$ <p>Start by multiplying the top equation by 2.</p>	$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$
$\begin{cases} 6x_1 + 2x_2 = 2 \\ 1x_1 + 2x_2 = 0 \end{cases}$ <p>Subtract the bottom equation from the top equation.</p>	$\begin{pmatrix} 6 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 6 & 2 & 2 \\ 1 & 2 & 0 \end{pmatrix}$
$\begin{cases} 5x_1 + 0x_2 = 2 \\ 1x_1 + 2x_2 = 0 \end{cases}$ <p>Multiply the bottom equation by 5.</p>	$\begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 5 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$
$\begin{cases} 5x_1 + 0x_2 = 2 \\ 5x_1 + 10x_2 = 0 \end{cases}$ <p>Subtract the top equation from the bottom equation.</p>	$\begin{pmatrix} 5 & 0 \\ 5 & 10 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 5 & 0 & 2 \\ 5 & 10 & 0 \end{pmatrix}$
$\begin{cases} 5x_1 + 0x_2 = 2 \\ 0x_1 + 10x_2 = -2 \end{cases}$ <p>Divide the top equation by 5 and the bottom by 10.</p>	$\begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 5 & 0 & 2 \\ 0 & 10 & -2 \end{pmatrix}$
$\begin{cases} 1x_1 + 0x_2 = \frac{2}{5} \\ 0x_1 + 1x_2 = -\frac{1}{5} \end{cases}$ <p>And we're done!</p>	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} \\ -\frac{1}{5} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & \frac{2}{5} \\ 0 & 1 & -\frac{1}{5} \end{pmatrix}$



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$$\begin{cases} 1x_1 + 0x_2 = \frac{2}{5} \\ 0x_1 + 1x_2 = -\frac{1}{5} \end{cases}$$

And we're done!

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} \\ -\frac{1}{5} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & \frac{2}{5} \\ 0 & 1 & -\frac{1}{5} \end{pmatrix}$$



SO YOU JUST
REWRITE THE
EQUATIONS AS
MATRICES AND
CALCULATE AS
USUAL?



WELL...

GAUSSIAN
ELIMINATION IS
ABOUT TRYING
TO GET THIS
PART HERE TO
APPROACH THE
IDENTITY MATRIX,
NOT ABOUT
SOLVING FOR
VARIABLES.



HMM...

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2012. The Manga Guide to Linear Algebra.
Starch Press (Manga guide series).

Matrix Inversion



LET'S TRY TO FIND AN INVERSE NEXT.


OKAY.

? PROBLEM

Find the inverse of the 2×2 matrix $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$

THINK ABOUT IT LIKE THIS.

SKRITCH
SKRITCH



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2012. The Manga Guide to Linear Algebra.
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We're trying to find the inverse of $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$



We need to find the matrix $\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$ that satisfies $\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$



or $\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}$ and $\begin{pmatrix} x_{12} \\ x_{22} \end{pmatrix}$ that satisfy $\begin{cases} \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_{12} \\ x_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases}$



We need to solve the systems $\begin{cases} 3x_{11} + 1x_{21} = 1 \\ 1x_{11} + 2x_{21} = 0 \end{cases}$ and $\begin{cases} 3x_{12} + 1x_{22} = 0 \\ 1x_{12} + 2x_{22} = 1 \end{cases}$

AH, RIGHT.



LET'S DO THE MATH.

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2012. The Manga Guide to Linear Algebra.
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SOLUTION

THE COMMON METHOD	THE COMMON METHOD EXPRESSED WITH MATRICES	GAUSSIAN ELIMINATION
$\begin{cases} 3x_{11} + 1x_{21} = 1 \\ 1x_{11} + 2x_{21} = 0 \end{cases} \quad \begin{cases} 3x_{12} + 1x_{22} = 0 \\ 1x_{12} + 2x_{22} = 1 \end{cases}$	$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$
Multiply the top equation by 2.	↓	↓
$\begin{cases} 6x_{11} + 2x_{21} = 2 \\ 1x_{11} + 2x_{21} = 0 \end{cases} \quad \begin{cases} 6x_{12} + 2x_{22} = 0 \\ 1x_{12} + 2x_{22} = 1 \end{cases}$	$\begin{pmatrix} 6 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 6 & 2 & 2 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$
Subtract the bottom equation from the top.	↓	↓
$\begin{cases} 5x_{11} + 0x_{21} = 2 \\ 1x_{11} + 2x_{21} = 0 \end{cases} \quad \begin{cases} 5x_{12} + 0x_{22} = -1 \\ 1x_{12} + 2x_{22} = 1 \end{cases}$	$\begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 & 0 & 2 & -1 \\ 1 & 2 & 0 & 1 \end{pmatrix}$
Multiply the bottom equation by 5.	↓	↓
$\begin{cases} 5x_{11} + 0x_{21} = 2 \\ 5x_{11} + 10x_{21} = 0 \end{cases} \quad \begin{cases} 5x_{12} + 0x_{22} = -1 \\ 5x_{12} + 10x_{22} = 5 \end{cases}$	$\begin{pmatrix} 5 & 0 \\ 5 & 10 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 0 & 5 \end{pmatrix}$	$\begin{pmatrix} 5 & 0 & 2 & -1 \\ 5 & 10 & 0 & 5 \end{pmatrix}$
Subtract the top equation from the bottom.	↓	↓
$\begin{cases} 5x_{11} + 0x_{21} = 2 \\ 0x_{11} + 10x_{21} = -2 \end{cases} \quad \begin{cases} 5x_{12} + 0x_{22} = -1 \\ 0x_{12} + 10x_{22} = 6 \end{cases}$	$\begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -2 & 6 \end{pmatrix}$	$\begin{pmatrix} 5 & 0 & 2 & -1 \\ 0 & 10 & -2 & 6 \end{pmatrix}$
Divide the top by 5 and the bottom by 10.	↓	↓
$\begin{cases} 1x_{11} + 0x_{21} = \frac{2}{5} \\ 0x_{11} + 1x_{21} = -\frac{1}{5} \end{cases} \quad \begin{cases} 1x_{12} + 0x_{22} = -\frac{1}{5} \\ 0x_{12} + 1x_{22} = \frac{3}{5} \end{cases}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & \frac{2}{5} & -\frac{1}{5} \\ 0 & 1 & -\frac{1}{5} & \frac{3}{5} \end{pmatrix}$
This is our inverse matrix; we're done!		

HUFF



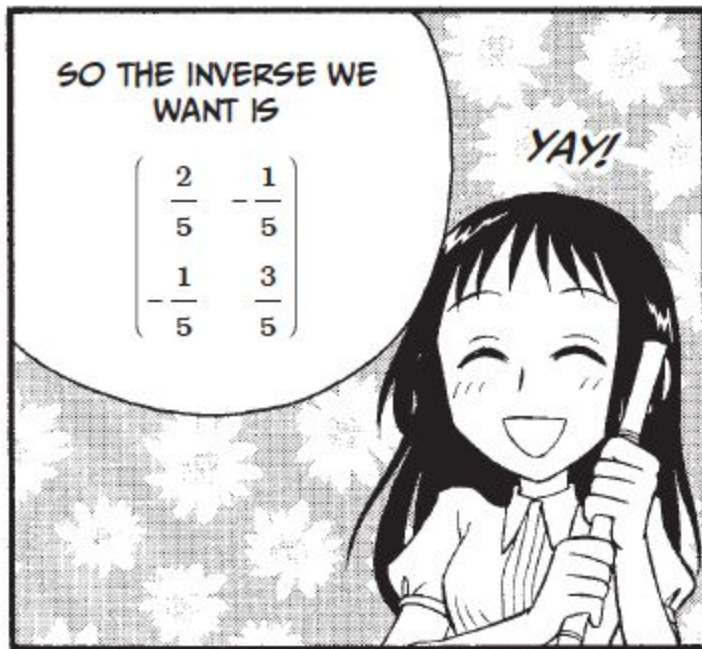
HUFF



DONE.



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LET'S MAKE SURE THAT THE PRODUCT OF THE ORIGINAL AND CALCULATED MATRICES REALLY IS THE IDENTITY MATRIX.



The product of the original and inverse matrix is

$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix} = \begin{pmatrix} 3 \cdot \frac{2}{5} + 1 \cdot \left(-\frac{1}{5}\right) & 3 \cdot \left(-\frac{1}{5}\right) + 1 \cdot \frac{3}{5} \\ 1 \cdot \frac{2}{5} + 2 \cdot \left(-\frac{1}{5}\right) & 1 \cdot \left(-\frac{1}{5}\right) + 2 \cdot \frac{3}{5} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of the inverse and original matrix is

$$\begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} \cdot 3 + \left(-\frac{1}{5}\right) \cdot 1 & \frac{2}{5} \cdot 1 + \left(-\frac{1}{5}\right) \cdot 2 \\ \left(-\frac{1}{5}\right) \cdot 3 + \frac{3}{5} \cdot 1 & \left(-\frac{1}{5}\right) \cdot 1 + \frac{3}{5} \cdot 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



IT SEEMS LIKE THEY BOTH BECOME THE IDENTITY MATRIX...

THAT'S AN IMPORTANT POINT: THE ORDER OF THE FACTORS DOESN'T MATTER. THE PRODUCT IS ALWAYS THE IDENTITY MATRIX! REMEMBERING THIS TEST IS VERY USEFUL. YOU SHOULD USE IT AS OFTEN AS YOU CAN TO CHECK YOUR CALCULATIONS.



Taken from: Takahashi S, Inoue I, Lindh F, C
2012. The Manga Guide to Linear Algebra.
Starch Press (Manga guide series).

BY THE WAY...



THE SYMBOL USED TO DENOTE INVERSE MATRICES IS THE SAME AS ANY INVERSE IN MATHEMATICS, SO...

THE INVERSE OF

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

IS WRITTEN AS

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}^{-1}$$



TO THE
POWER OF
MINUS ONE,
GOT IT.

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2012. The Manga Guide to Linear Algebra.
Starch Press (Manga guide series).

Matrix Decomposition

Various types of decomposition.
Some examples:

- LU decomposition
- QR decomposition



LU Decomposition Example

<https://youtu.be/UqemQTirijg>



Please review these operations

- We will revisit these operations in the succeeding weeks, so be sure you are familiar with them again

