CoE 163

Computing Architectures and Algorithms

01a: Algorithms Review

REMEMBER EEE 121?

Data structures and **algorithms** are key in solving any computer engineering problem.

Knowledge of these basic concepts enable you to solve large real-world problems.

BASIC DATA STRUCTURES

- Basic
    - Numbers
    - Strings
    - Sets
- Linear
    - Arrays, linked lists
    - Stacks, queues
- Graph
    - Adjacency matrix
    - Adjacency list
    - Disjoint set

BASIC DATA STRUCTURES

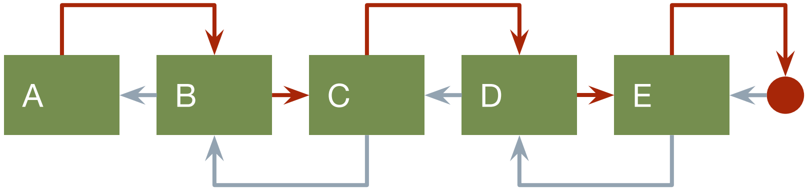
- Trees, heaps
  - Binary tree (AVL, red-black)
  - String trees (trie)
- Geometry
  - Point pairs
  - Polygon list

DATA STRUCTURES: LINEAR

Array: elements of usually same type arranged linearly

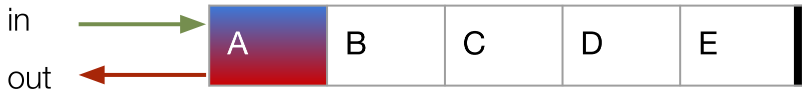
A	B	C	D	E

Linked list: a loosely-connected array



DATA STRUCTURES: LINEAR

Stack: last in, first out; single-ended array



Queue: first in, first out; double-ended array

DATA STRUCTURES: LINEAR

- Arrays are useful for fixed and arranged things
    - AA battery chargers
    - Piano keys
- Linked lists are useful for things where middle elements can change
    - Clinic appointments with cancellations
    - Word editing (letter insertion/deletion)

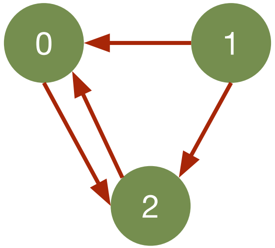
DATA STRUCTURES: LINEAR

- Stacks are useful for things that need stacking - usually vertical
  - Box stacking in warehouses
  - Tetris
- Queues are useful for things that fall in line - usually horizontal
  - Queueing systems in fast food
  - Groceries sorted by expiry date
- A stack and a queue in one is called a **deque** (double-ended queue)

DATA STRUCTURES: GRAPH

Adjacency matrix: 2D array with indices as the two nodes and value the weight or interconnection flag

		destination		
		node 0	node 1	node 2
origin	node 0	0	0	1
	node 1	1	0	1
	node 2	1	0	0

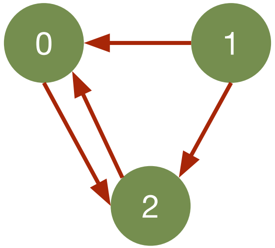


- Rows correspond to origin node and columns the destination node
  - Can be reversed depending on how you code the graph
  - An undirected graph can be represented as a symmetric matrix
- If an element along the diagonal is nonzero, there is an edge to the element itself

DATA STRUCTURES: GRAPH

Adjacency list: Array of variable-length arrays listing neighbors of a node

node	neighbors	
node 0	2	
node 1	0	2
node 2	0	

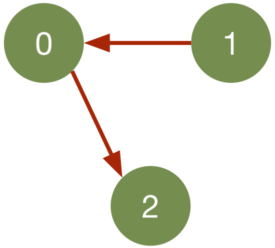


- Saves space as it does not allocate a vxv matrix (v the number of nodes)
- An edge to a node itself can be represented by listing itself in its adjacency list

DATA STRUCTURES: GRAPH

Disjoint set: Array with indices as node labels and value denoting which node is its parent

	node		
	node 0	node 1	node 2
parent	2	0	-1



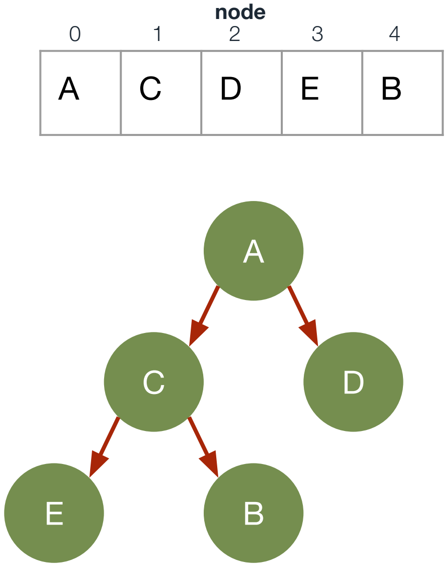
- This is actually a <u>set data structure</u>, but usually comes up in graphs
- Disjoint sets do not have cycles and have only <u>one parent</u>
- Traversal is recursive and can be implemented efficiently if paths are compressed

DATA STRUCTURES: GRAPH

- Adjacency matrices are useful for dense and small graphs
  - "Flow Free" game
- Adjacency lists are useful for sparse and large graphs
  - Road networks
- Disjoint sets are useful for child-parent-like relationships
  - Family trees

DATA STRUCTURES: TREE

Binary tree: Tree that has at most two children



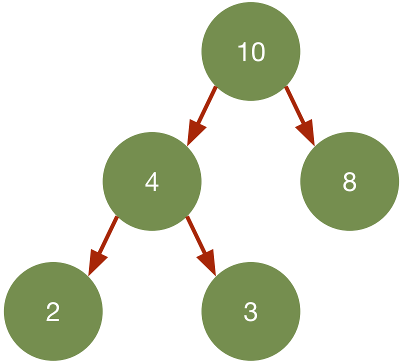
- There are different kinds of binary trees
  - AVL, red-black, splay…
- Balancing is important to ensure efficient traversal and mutation
- Implement as a graph, linked “list”, or 1D array
  - Linked “list” consists of nodes, with each tracking the left and right subtrees
  - 1D array arranged as breadth-first traversal

DATA STRUCTURES: TREE

Heap: Tree that satisfies the heap property (parent root has higher/lower value than children)

node

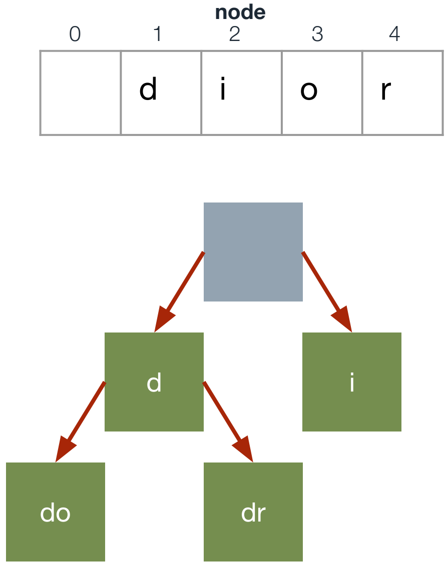
0	1	2	3	4
10	4	8	2	3



- Max heap has the highest-valued node at the root
- Can be stored as a 1D array the same as a binary tree
- Balancing is important to maintain the heap property

DATA STRUCTURES: TREE

Trie (Prefix tree): Tree that locates specific keys within a set



- A node is defined by its parent prefix and its value concatenated
- Can be stored as a 1D array with the suffix as value
- Children of leaf nodes need to be represented with a symbol to denote end of trie

DATA STRUCTURES: TREE

- Binary trees are useful for balanced matching and searching
  - Parentheses matching
- Heaps are useful for maintaining order while mutating data
  - Senior citizen lane in groceries
- Tries are useful for matching and finding
  - String searching

BASIC ALGORITHMS

- Graph theory
  - Traversal and shortest paths
  - Minimum spanning tree
- Problem solving paradigms
  - Complete search and recursion
  - Divide and conquer
  - Dynamic programming/greedy

BASIC ALGORITHMS

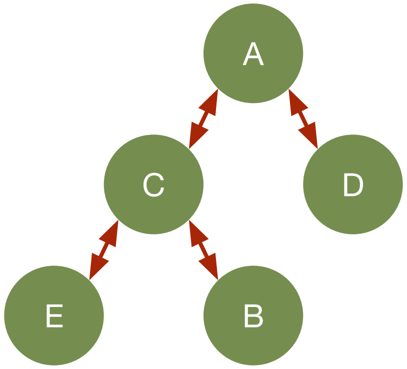
- Math and geometry
  - Probability and statistics
  - Plane/analytic/spherical geometry
- String processing
  - String matching
  - Trees, tries, and arrays
- Data processing
  - Sorting
  - Filter and transformation

ALGORITHMS: GRAPH TRAVERSAL

Graph traversal: Search by visiting a node and its neighbors systematically

Traversal order

- DFS: A-C-E-B-D
- BFS: A-C-D-E-B



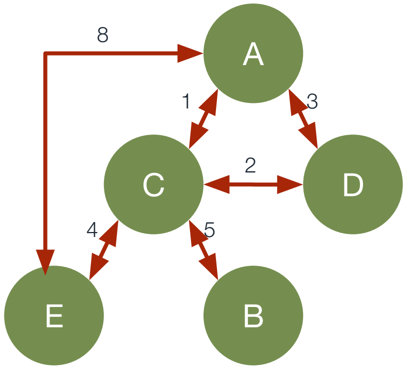
- Depth-first search (DFS): visit the deepest part of a path, then backtrack
  - Uses a stack to track nodes being visited
- Breadth-first search (BFS): visit by layer
  - Uses a queue to track nodes being visited
- Traversal can be modified to determine shortest path between two nodes

ALGORITHMS: SHORTEST PATH

Shortest path: Find path between two nodes that has the minimum weight

Some shortest paths

- A to D: costs 3 (A-D)
- A to E: costs 5 (A-C-E)



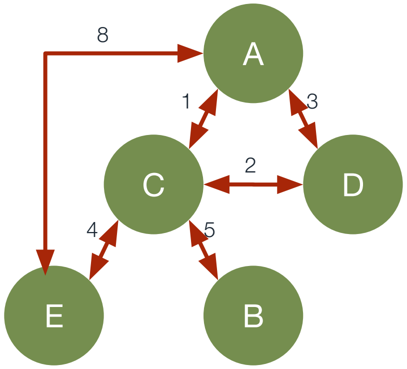
- Dijkstra's: visit and "relax" edges to find minimum-weighted path
  - A priority queue can be used to pick which nodes to visit first
- Bellman-Ford: similar to Dijkstra's but works on negative weights
  - Uses dynamic programming to "relax' edges

ALGORITHMS: MINIMUM SPANNING TREE

Minimum spanning tree: Find set of edges that cumulatively have the total minimum weight and still connects all nodes

Minimum edges needed

- With weights 1, 2, 4, 5



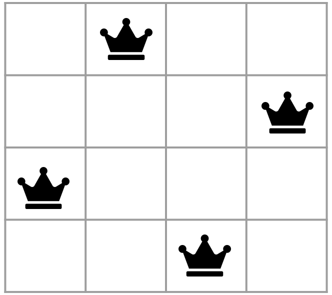
- Kruskal: Sort edges from the lowest weight and get those on top if the two nodes are not connected yet
- Prim: Select a random node and pick a connecting edge with the lowest weight. Collect edges from the resulting connected node and repeat choosing of edges *among all the connected nodes* in such fashion.

ALGORITHMS: PROBLEM SOLVING

Complete search: Iterate through all possibilities of a solution systematically

Put queens on a grid where they do not threaten each other

- Put queens by row, taking care to put them in separate columns
- Check for threats at the diagonal and backtrack to previous layout if there are



- Brute force: Create nested loops or recursions to explore all possibilities
- A*: Explore nodes with the lowest cost first
- Graph traversal: Reform problem into a graph problem and traverse through all possibilities

ALGORITHMS: PROBLEM SOLVING

Divide and conquer: Recurse through a problem by splitting it into n similar problems and consolidating the solutions

You have a cat of length a. Find two cats on a row of cats ordered from shortest length that is a little longer and little shorter than yours.

- Use binary search to find the floor and ceiling lengths

row of cats

1	2	3	5	7	10	12

Your cat is of length 8
Go to segments 5-12, 5-7, 10-12
Closest lengths are 7 and 10

- Binary search on a tree is an example
- Bisection method is useful in arriving at a numerical solution

ALGORITHMS: PROBLEM SOLVING

Dynamic programming: Prune complete search by observing recursion leading to an optimal solution

Determine grouping of matrix chain multiplications that will yield the smallest number of operations

- A: 3x2, B: 2x5, C: 5x4; E = ABC
- Group matrices like complete search
- Overwrite saved solution if it is smaller

from \ until	A	B	C
A	0	30	64
B		0	40
C			0

- Top-down: Recurse from the top and parts of the solution for later rebuilds
- Bottom-up: Build up to the solution from base cases
- Build order is important!
- Solution configuration can be recovered by saving previous iterations

ALGORITHMS: PROBLEM SOLVING

Greedy: Get what is best at the moment

Buy two take one free promo! Find the maximum discount you can get given your basket items.

- Go to the counter with the three most expensive items on your basket every time

Saved 9!

- Special case of dynamic programming that satisfies the greedy property
- Although it does not work all the time, it can yield fast and slightly suboptimal solutions
- When in doubt, use dynamic programming instead

ALGORITHMS: MATH AND GEOMETRY

Basic arithmetic: Remember elementary axioms, factors, etc.

Three friends share a garden - one worked A hours and another worked B hours to clean up the whole garden. The third friend paid D dollars. How much should A get?

$$S_A = \left(A - \frac{A+B}{3}\right) \frac{D}{\frac{A+B}{3}}$$

A, B, and C have equal shares. A and B clean up their respective areas plus extra time that they give up to clean C's area.

- Radix/Base conversion
- Numerical pattern finding
- Fractions
- Logarithms and exponents
- Prime numbers
- Modular arithmetic
- Euclidean algorithm

ALGORITHMS: MATH AND GEOMETRY

Probability and statistics: Apply basic probability axioms and combinatorics

Monty Hall problem - find the chance of winning when you switch to another door

doors

?	goat	?

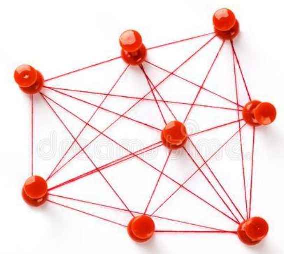
If you stayed with your original choice, it's as if you just opened that door straight away, so the chance is ⅓.

The chance if switching is therefore ⅔.

- Permutations and combinations
- Bayes' theorem, conditional probabilities
- Binomial, Catalan, Fibonacci numbers

ALGORITHMS: MATH AND GEOMETRY

Geometry: Apply 2D and 3D geometry theorems and conjectures



- Line and plane intersections
- Area, perimeter, volume
- Convex hull
- Point inside polygon
- Be careful of numerical errors when using floating-point!

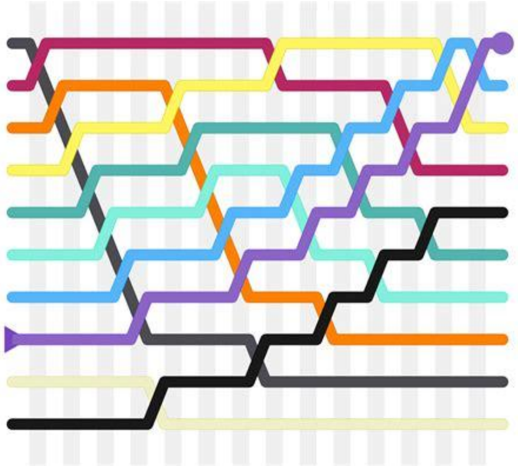
ALGORITHMS: STRINGS

String matching, processing, and manipulation

- Knuth-Morris-Pratt algorithm
- String alignment
- Suffix trie, prefix tree, arrays

ALGORITHMS: DATA PROCESSING

Apply algorithms to sort, filter, and transform data



- Bubble, insertion, selection sorts
- Priority queue
- Summation and production
- Bit masking
- Character to ASCII value

TIPS

- Learn new algorithms and data structures
- Be exposed to a lot of known CS problems
- Practice by trying out online judges, solving some problems, and getting used to input/output formatting

CoE 163

Computing Architectures and Algorithms

01a: Algorithms Review