CoE 163

Computing Architectures and Algorithms

Solving problems using linear algebra

Main reference

fast.ai course: Computational Linear Algebra

Written: 17 Jul 2017 by *Rachel Thomas*

Lecture 1: “Why are we here?”

https://nbviewer.jupyter.org/github/fastai/numerical-linear-algebra/blob/master/nbs/1.%20Why%20are%20we%20here.ipynb#Answer

What is Linear Algebra useful for?

- Linear algebra allows us to solve many problems in science and engineering. For example:
  - Reaction stoichiometry (balancing equations)
  - Electronic circuit analysis (current flow in networks)
- If we can perform linear algebra operations using a computer to handle larger data, it can be useful for many interesting applications
- These slides will demonstrate some of the problems we can solve using linear algebra

Two key types of matrix computation

(Hope you reviewed the matrix operations!)

1. Matrix and tensor products
2. Matrix decompositions

The top 10 algorithms of science and engineering during the 20th century

In putting together this issue of *Computing in Science & Engineering*, we knew three things: it would be difficult to list just 10 algorithms; it would be fun to assemble the authors and read their papers; and, whatever we came up with in the end, it would be controversial. We tried to assemble the 10 algorithms with the greatest influence on the development and practice of science and engineering in the 20th century. Following is our list (here, the list is in chronological order; however, the articles appear in no particular order):

- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

Let’s look at some practical applications of using linear algebra to solve problems

Matrix-vector multiplication

The matrix shown gives the probabilities of moving from 1 health state to another in 1 year. If the current health states for a group are:

- 85% asymptomatic
- 10% symptomatic
- 5% AIDS
- 0% death

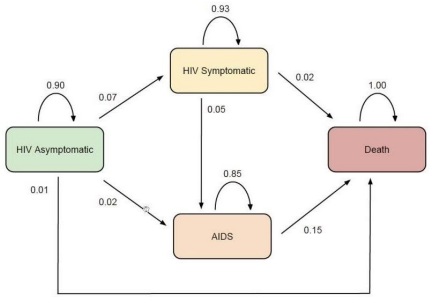
what will be the % in each health state in 1 year?

HIV/AIDS Example

This stochastic matrix:

$$\begin{pmatrix} 0.90 & 0.07 & 0.02 & 0.01 \\ 0 & 0.93 & 0.05 & 0.02 \\ 0 & 0 & 0.85 & 0.15 \\ 0 & 0 & 0 & 1.00 \end{pmatrix}$$

corresponds to:



- You can write simple code to solve this (or do it by hand)
- Can we use a similar technique to make projections related to the current pandemic?

Matrix-Matrix Multiplication

Three people denoted by P_1, P_2, P_3 intend to buy some rolls, buns, cakes and bread. Each of them needs these commodities in differing amounts and can buy them in two shops S_1, S_2. Which shop is the best for every person P_1, P_2, P_3 to pay as little as possible? The individual prices and desired quantities of the commodities are given in the following tables:

Demanded quantity of foodstuff:

	roll	bun	cake	bread
P_1	6	5	3	1
P_2	3	6	2	2
P_3	3	4	3	1

Prices in shops S_1 and S_2:

	S_1	S_2
roll	1.50	1.00
bun	2.00	2.50
cake	5.00	4.50
bread	16.00	17.00

For example, the amount spent by the person P_1 in the shop S_1 is:

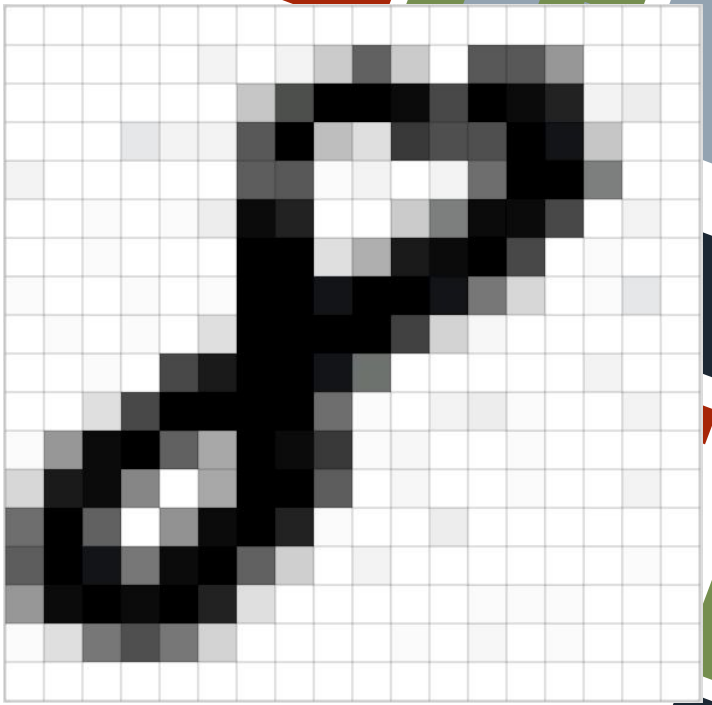
$$6 \cdot 1.50 + 5 \cdot 2 + 3 \cdot 5 + 1 \cdot 16 = 50$$

and in the shop S_2 :

$$6 \cdot 1 + 5 \cdot 2.50 + 3 \cdot 4.50 + 1 \cdot 17 = 49,$$

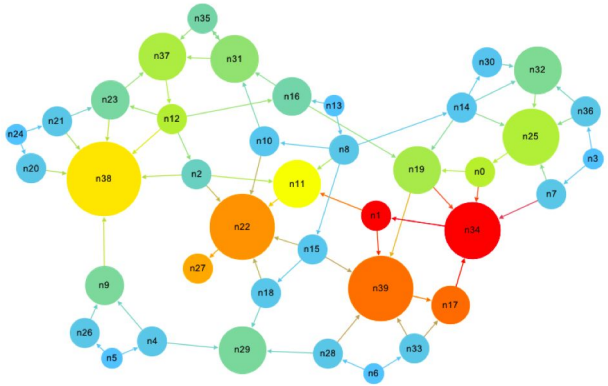
Images can be represented using matrices

- Image is a handwritten number ‘8’
- Elements in the matrix represent the shade/darkness of each pixel



Eigen Decomposition for Google's PageRank Algorithm

This is the algorithm that makes Google's search find the 'most relevant' pages



Check out the first part of this discussion on PageRank

To get an idea how matrices are useful in the original Google search algorithm, check this out: https://jeremykun.com/2011/06/18/googles-pagerank-a-first-attempt/

(Of course, you are free to read through the rest of the blog entries

Key takeaways

- Linear algebra has many useful applications
- For many of these applications, it will need to handle larger sets of data -> impractical by hand, better using a computer
- Creating algorithms for linear algebra operations is thus very useful
- Next set of slides: How do we make sure these algorithms are accurate and efficient?